

**Domain: Number and Operations—Fractions**

In grade three students develop an understanding of fractions as numbers, beginning with unit fractions by building on the idea of partitioning a whole into equal parts. Student proficiency with fractions is essential for success in more advanced mathematics such as percentages, ratios and proportions, and in algebra at later grades.

Number and Operations—Fractions <sup>8</sup>	3.NF
<p><b>Develop understanding of fractions as numbers.</b></p> <ol style="list-style-type: none"> <li>1. Understand a fraction <math>1/b</math> as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</li> <li>2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.               <ol style="list-style-type: none"> <li>a. Represent a fraction <math>1/b</math> on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>1/b</math> and that the endpoint of the part based at 0 locates the number <math>1/b</math> on the number line.</li> <li>b. Represent a fraction <math>a/b</math> on a number line diagram by marking off <math>a</math> lengths <math>1/b</math> from 0. Recognize that the resulting interval has size <math>a/b</math> and that its endpoint locates the number <math>a/b</math> on the number line.</li> </ol> </li> </ol>	

In grades one and two, students partitioned circles and rectangles into two, three, and four equal shares and used fraction language (e.g., halves, thirds, half of, a third of). In grade three, students begin to enlarge their concept of number by developing an understanding of fractions as numbers (Adapted from PARCC 2012).

Grade three students understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts and the fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ . **(3.NF.1 ▲)**.

[Note: Sidebar]

**Focus, Coherence, and Rigor:**

When working with fractions, two main ideas should be emphasized:

- Specifying the whole

<sup>8</sup> Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

- Explaining what is meant by “equal parts”
- Student understanding of fractions hinges on understanding these ideas.

Students build on the idea of *partitioning* or dividing a whole into equal parts to understand fractions. Students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts (the number of equal parts becomes the denominator) and taking one of those parts. An important goal is for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers. Students make the connection that just as every whole number is obtained by combining a sufficient number of 1s; every fraction is obtained by combining a sufficient number of unit fractions (Adapted from Progressions 3-5 NF 2012). They explore fractions first using concrete models such as fraction bars and geometric shapes, which will culminate in understanding fractions on the number line.

#### Examples:

Show the fraction  $\frac{1}{4}$  by folding the piece of paper into equal parts.

“I know that when the number on the bottom is 4, I need to make four equal parts. By folding the paper in half once and then again, I get four parts and each part is equal. Each part is worth  $\frac{1}{4}$ .”



Shade  $\frac{3}{4}$  using the fraction bar you created.

“My fraction bar shows fourths. The 3 tells me I need three of them, so I’ll shade them. I could have shaded any three of them and I would still have  $\frac{3}{4}$ .”

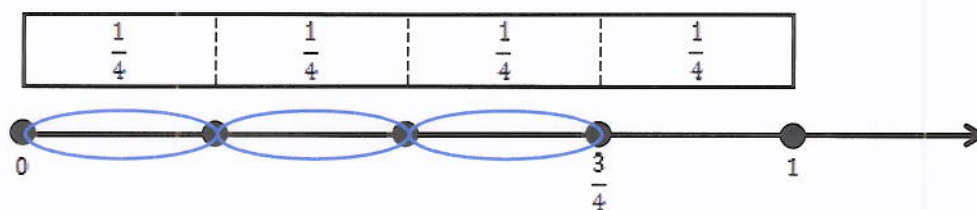




1/b when locating fractions on the number line. Notice the difference between how the fraction bar and number line are labeled in the example shown below (3.NF.2a-b).

**Example (Representing Fractions on the Number Line):** Use your fraction bar and the number line given to locate the fraction  $\frac{3}{4}$ . Explain how you know your mark is in the right place.

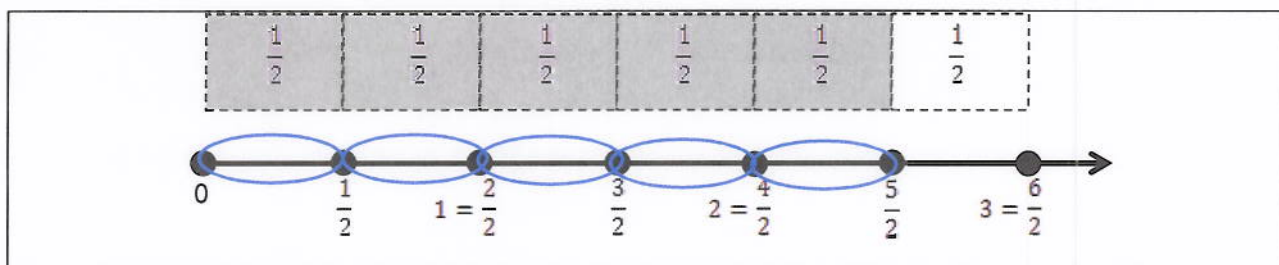
**Solution:** "When I use my fraction strip as a measuring tool, it shows me how to divide the unit interval into four equal parts (since the denominator is 4). Then I start from the mark that has '0' and I measure off three pieces of  $\frac{1}{4}$  each. I circled the pieces to show that I marked three of them. This is how I know I have marked  $\frac{3}{4}$ ."



Third grade students need opportunities to place fractions on a number line and understand fractions as a related component of the ever-expanding number system. The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so is  $\frac{5}{3}$  the point obtained by marking off 5 times the length of a different interval as the basic unit of length, namely the interval from 0 to  $\frac{1}{3}$ .

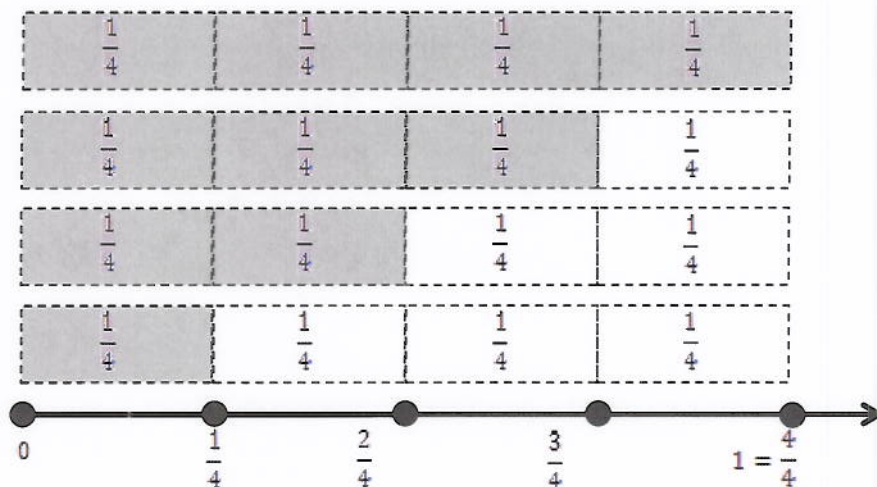
**Fractions Greater Than One.** Note that the standards do not distinguish fractions greater than one as being "improper fractions." Fractions greater than one, such as  $\frac{5}{2}$ , are simply numbers in themselves and are constructed in the same way as other fractions.

Thus, to construct  $\frac{5}{2}$ , we might use a fraction strip as a measuring tool to mark off lengths of  $\frac{1}{2}$ . Then we count five of those halves, to get  $\frac{5}{2}$ .



320  
 321 Students recognize that when examining fractions with common denominators, the  
 322 wholes have been divided into the same number of equal parts, so the fraction with the  
 323 larger numerator has the larger number of equal parts. Students develop an  
 324 understanding of the numerator and denominator as they label each fractional part  
 325 based on how far it is from zero to the endpoint. (MP.7)

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### Number and Operations—Fractions<sup>9</sup>

3.NF

#### Develop understanding of fractions as numbers.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
  - a. Understand two fractions as equivalent (equal) if they are the same size, or the same end point on a number line.
  - b. Recognize and generate simple equivalent fractions, e.g.,  $1/2 = 2/4$ ,  $4/6 = 2/3$ . Explain why the fractions are equivalent, e.g., by using a visual fraction model.
  - c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form  $3 = 3/1$ ; recognize that  $6/1 = 6$ ; locate  $4/4$  and 1 at

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*the same point on a number line diagram.*

- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a fraction model.

329  
330 Students develop an understanding of fractions as they use visual models and a  
331 number line to represent, explain, and compare unit fractions, equivalent fractions (e.g.,  
332  $\frac{1}{2} = \frac{2}{4}$ ), whole numbers as fractions (e.g.,  $3 = \frac{3}{1}$ ), and fractions with the same numerator  
333 (e.g.,  $\frac{4}{3}$  and  $\frac{4}{6}$ ) or the same denominator (e.g.,  $\frac{4}{8}$  and  $\frac{5}{8}$ ). **(NF.2-3 ▲)**.

334  
335 Students develop an understanding of order in terms of position on a number line.  
336 Given two fractions—thus two points on the number line—students understand that the  
337 one to the left is said to be smaller, and the one to the right is said to be larger (Adapted  
338 from Progressions 3-5 NF 2012).

339  
340 Students learn that when comparing fractions they need to look at the size of the parts  
341 and the number of the parts. For example,  $\frac{1}{8}$  is smaller than  $\frac{1}{2}$  because when 1 whole is  
342 cut into 8 pieces, the pieces are much smaller than when 1 whole of the same size is  
343 cut into 2 pieces.

344  
345 To compare fractions that have the same numerator but different denominators,  
346 students understand that each fraction has the same number of equal parts but the size  
347 of the parts is different. They can infer that the same number of smaller pieces is less  
348 than the same number of bigger pieces (Adapted from Arizona 2012 and KATM 3<sup>rd</sup>  
349 FlipBook 2012).

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351 Students develop an understanding of equivalent fractions as they compare fractions  
352 using a variety of visual fraction models and justify their conclusions **(MP.3)**. Through  
353 opportunities to compare fraction models with the same whole divided into different

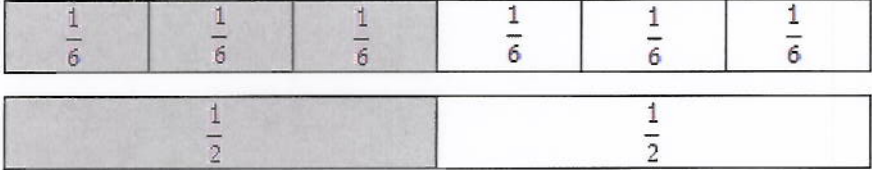
354 numbers of pieces, students identify fractions that show the same amount or name the  
 355 same number, and learn that they are equal (or equivalent).

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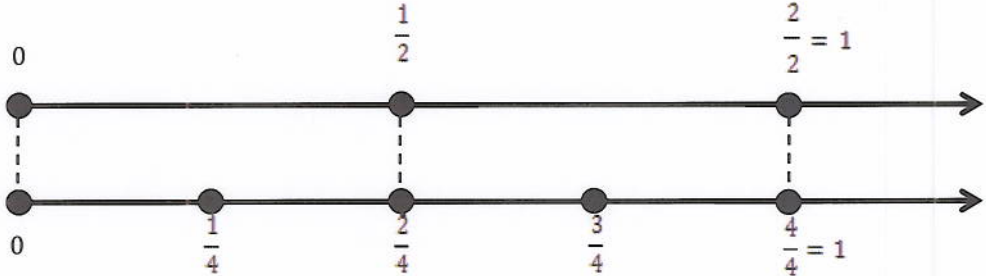
357 (Adapted from Progressions 3-5 NF 2012)

**Basic Fraction Equivalence Using Models**

Using Fraction Bars:



Using a number line:



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**Some important concepts related to understanding fractions include:**

- Fractional parts must be equal-sized
- The number of equal parts tells how many make a whole
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- The size of the fractional part is relative to the whole.
- When a shape is divided into equal parts, the denominator represents the number of equal parts in the whole and the numerator of a fraction is the count of the demarcated congruent or equal parts in a whole (e.g.,  $\frac{3}{4}$  means that there are 3 one-fourths or 3 out of 4 equal parts).
- Common benchmark numbers such as 0,  $\frac{1}{2}$ ,  $\frac{3}{4}$  and 1 can be used to determine if an unknown fraction is greater or smaller than a benchmark fraction.

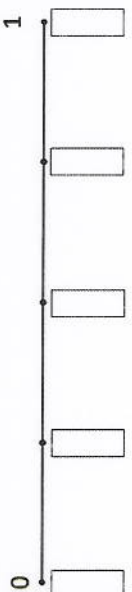
359 (Adapted from Arizona 2012 and KATM 3<sup>rd</sup> FlipBook 2012)

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361 The *Fractions Progression Module* provides an overview of understanding fractions and  
362 is available at [http://www.illustrativemathematics.org/pages/fractions\\_progression](http://www.illustrativemathematics.org/pages/fractions_progression)  
363 (Illustrative Mathematics 2013).

364  
365 Following is a sample classroom activity that connects the Standards for Mathematical  
366 Content and Standards for Mathematical Practice.



Standards	Explanations and Examples
<p><b>3.NF.1:</b> Understand a fraction <math>1/b</math> as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</p> <p><b>3.NF.2:</b> Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction <math>1/b</math> on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>1/b</math> and that the endpoint of the part based at 0 locates the number <math>1/b</math> on the number line.</p> <p>b. Represent a fraction <math>a/b</math> on a number line diagram by marking off the lengths <math>1/b</math> from 0. Recognize that the resulting interval has size <math>a/b</math> and that its endpoint locates the number <math>a/b</math> on the number line.</p> <p><b>3.NF.3:</b> Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>b. Recognize and generate simple equivalent fractions, (e.g. <math>1/2=2/4</math>, <math>4/6=2/3</math>). Explain why the fractions are equivalent, e.g. by using a visual fraction model.</p>	<p><b>Task: The Human Fraction Number Line Activity.</b> In this activity, the teacher posts a long sheet of paper on a wall of the classroom to act as a number line, with 0 marked at one end and 1 marked at the other. Gathered around the wall, groups of students will be given cards with different sized fractions indicated on them (e.g. <math>0 \frac{1}{4}</math>, <math>\frac{2}{4}</math>, <math>\frac{3}{4}</math>, <math>\frac{4}{4}</math>) and will be asked to locate themselves approximately along the number line. Depending on the size of the class and the length of the number line, fractions with denominators 2, 3, 4, 6, and 8 can be used. The teacher can ask students to explain to each other why their placements are correct or incorrect, emphasizing that the students with cards marked in fourths, say, have divided the number line into four equal parts. Furthermore, a student with the card <math>a/b</math> is standing in the correct place if they represent a length of size <math>1/b</math> from 0 on the number line.</p> <p>As a follow-up activity, teachers can give students several unit number lines that are marked off into equal parts but that are unlabeled. Students are required to fill in the labels on the number lines. An example is shown here:</p>  <p><b>Classroom Connections:</b> There are several big ideas included in this activity. One is that when talking about fractions as points on a number line, the whole is represented by the <i>length</i> or amount of distance from 0 to 1. By physical requiring students to line up in the correct places on the number line, the idea of partitioning this distance into equal parts is emphasized. In addition, other students can physically mark off the placement of fractions by starting from 0 and walking the required number of lengths <math>1/b</math> from 0; for example, with students placed at the locations for sixths, another student can start at 0 and walk off a distance of <math>5/6</math>. As an extension, teachers can have students mark off equivalent fraction distances, such as <math>1/2</math>, <math>2/4</math>, and <math>3/6</math>, and can have a discussion as to why those fractions represent the same amount.</p> <p><b>Connecting to the Standards for Mathematical Practice:</b></p> <p>(MP.2) Students reason quantitatively as they determine why a placement was correct or incorrect and by assigning a fractional value to a distance.</p> <p>(MP.4) Students are using the number line model for fractions. While not an application of mathematics to a real-world situation in the sense of true modeling, it is an appropriate use of modeling for the grade level.</p> <p>(MP.8) Students see repeated reasoning in dividing up the number line into equal parts, though of various sizes, and form the basis for how they would place fifths, tenths, and other fractions.</p>